Math 1A: Calculus **Instructor: Alexander Paulin**

Quiz 3: Limits

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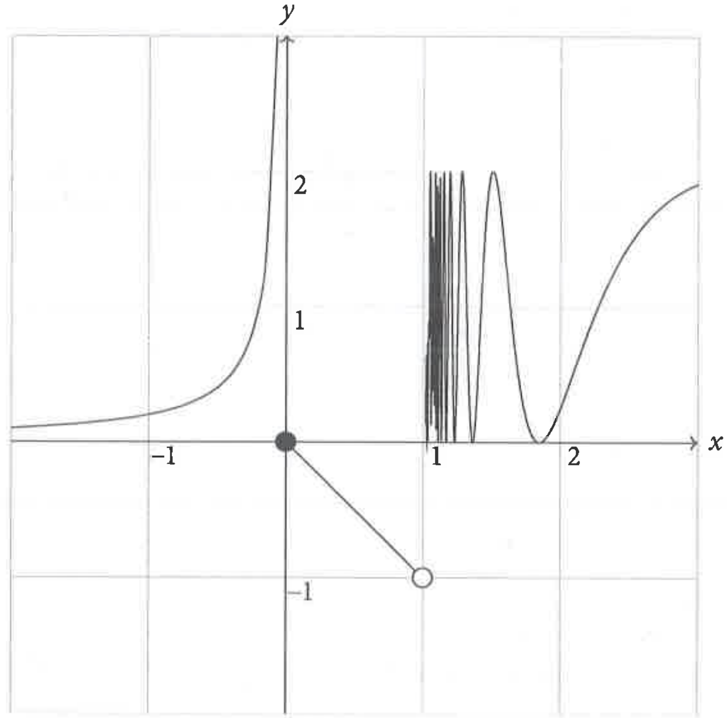
Problem 1 (5 points, 1 point per question). This problem asks you to compute limits visually. The graph of $y = f(x)$ is shown below.

Compute each of the following limits. (Give a number if the limit exists, or write ∞, $-\infty$, or DNE as appropriate if the limit does not exist.)

- $\lim_{x\to 0^-} f(x) =$ ∞
- $\lim_{x\to 0^+} f(x) =$ 0
- $\lim_{x\to 0} f(x) =$ DNE
- $\lim_{x\to 1^-} f(x) =$ -1
- $\lim_{x\to 1^+} f(x) =$ DNE



Problem 2 (5 points). Compute the limit

$$\lim_{x\to 3}\frac{x^2-5x+6}{x^2-2x-3}.$$

Please carefully justify all your steps.

$$\lim_{x\to 3}\frac{x^2-5x+6}{x^2-2x-3} = \lim_{x\to 3}\frac{(x-3)(x-2)}{(x-3)(x+1)}$$

$$= \lim_{x\to 3}\frac{x-2}{x+1}$$ because $\frac{(x-3)(x-2)}{(x-3)(x+1)} = \frac{x-2}{x+1}$ when $x \neq 3$

$$= \frac{3-2}{3+1}$$ because this is a rational function defined on an interval containing $x=3$, such as $(2,4)$

$$= \boxed{\frac{1}{4}}$$

(Continued on back of sheet.)

Problem 3 (5 points). Use the Squeeze Theorem to show that

$$\lim_{x\to 0} x^3 \cos\left(\frac{1}{x}\right) = 0.$$

$f(x) = x^3 \cos\left(\frac{1}{x}\right)$

$-1 \le \cos\left(\frac{1}{x}\right) \le 1$ $(x \neq 0)$ because $\cos(x)$ has range $[-1,1]$.

$$\left.\begin{aligned} -x^3 \le x^3 \cos\left(\tfrac{1}{x}\right) \le x^3 \quad (x>0) \\ -x^3 \ge x^3 \cos\left(\tfrac{1}{x}\right) \ge x^3 \quad (x<0) \end{aligned}\right\} \text{2 cases depending on sign of } x^3.$$

Let $g(x) = \begin{cases} x^3 & x<0 \\ 0 & x=0 \\ -x^3 & x>0 \end{cases}$, $h(x) = \begin{cases} -x^3 & x<0 \\ 0 & x=0 \\ x^3 & x>0 \end{cases}$

(can also write as $-|x^3|$) (can also write as $|x^3|$).

Then $g(x) \le f(x) \le h(x)$ when $x \neq 0$. Also: $\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} (-x^3) = 0$

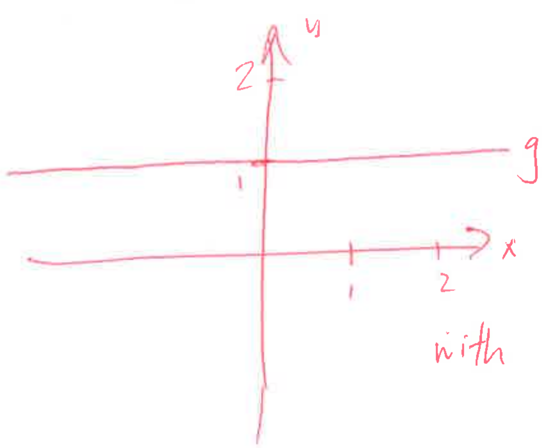
..... similar computations show $\lim_{x\to 0} g(x) = 0$ and $\lim_{x\to 0} h(x) = 0$.

Apply Squeeze Thm.

Optional Problem (this is not worth any points). Consider the following claim:

"If $\lim_{x\to a} g(x) = b$ and $\lim_{x\to b} f(x) = c$, then $\lim_{x\to a} f(g(x)) = c$."

While this claim may sound reasonable, it is actually *false*. Come up with a counterexample: specify functions f, g and numbers a, b, c such that the hypotheses are satisfied but the conclusion is untrue.



with $a=1$
$b=1$
$c=0$